Four/ Six marks Questions

1   Using properties of determinants, prove that

$$\begin{vmatrix} \alpha & \alpha^2 & \beta + \gamma \\ \beta & \beta^2 & \gamma + \alpha \\ \gamma & \gamma^2 & \alpha + \beta \end{vmatrix} = (\beta - \gamma)\ (\gamma - \alpha)\ (\alpha - \beta)\ (\alpha + \beta + \gamma)$$

2   Using properties of determinants, prove that

$$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)\ (y-z)\ (z-x)\ (xy+yz+zx)$$

3   Using properties of determinants, prove that

$$\begin{vmatrix} \alpha & \alpha^2 & \beta + \gamma \\ \beta & \beta^2 & \gamma + \alpha \\ \gamma & \gamma^2 & \alpha + \beta \end{vmatrix} = (\beta - \gamma)\ (\gamma - \alpha)\ (\alpha - \beta)\ (\alpha + \beta + \gamma)$$

4   By using properties show that

$$\begin{vmatrix} a^2 + 1 & ab & ac \\ ab & b^2 + 1 & bc \\ ca & cb & c^2 + 1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

5   Using properties of determinants, prove that

$$\begin{vmatrix} b + c & c + a & a + b \\ c + a & a + b & b + c \\ a + b & b + c & c + a \end{vmatrix} = -(a+b+c)\ [(a-b)^2+(b-c)^2+(c-a)^2]$$

6   By using properties of determinants show that

$$\begin{vmatrix} 1 + x & 1 & 1 \\ 1 & 1 + y & 1 \\ 1 & 1 & 1 + z \end{vmatrix} = xyz + xy + yz + zx$$

7   Show that $\begin{vmatrix} x+1 & x+2 & x+a \\ x+2 & x+3 & x+b \\ x+3 & x+4 & x+c \end{vmatrix} = 0$, where a, b, c are in AP.

8 Find the maximum value of

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+\sin x & 1 \\ 1 & 1 & 1+\cos x \end{vmatrix}$$

9 If a, b, c are real numbers and $\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0.$

Show that either $a+b+c=0$ or $a=b=c$

10 Show that $\begin{vmatrix} x & x+y & x+2y \\ x+2y & x & x+y \\ x+y & x+2y & x \end{vmatrix} = 9y^2(x+y)$

11 Using properties of determinants, prove that

$$\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3 + b^3 + c^3 - 3abc$$

12 Using properties of determinants, prove that

$$\begin{vmatrix} a^2 + 2a & 2a+1 & 1 \\ 2a+1 & a+2 & 1 \\ 3 & 3 & 1 \end{vmatrix} = (a-1)^3$$

13 Using properties of determinants prove that

$$\begin{vmatrix} 1 & a^2 + bc & a^3 \\ 1 & b^2 + ca & b^3 \\ 1 & c^2 + ab & c^3 \end{vmatrix} = -(a-b)(b-c)(c-a)(a^2 + b^2 + c^2)$$

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